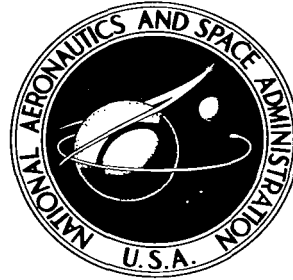


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FIELD EMISSION CHARGING OF METALLIC COLLOIDS

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SUMMARY

With a free-electron model of metallic colloids, the charging time in an applied electric field is calculated for the alkali-group elements. A theoretical thin-wire ionizer is discussed with regard to required operating potentials, necessary geometrical dimensions, and current-density capabilities.

INTRODUCTION

The theoretical improvement of electrostatic-thruster efficiency resulting from the use of charged particles of large mass has been recognized for some time (ref. 1). Current studies on heavy-particle thrusters have developed in two areas. One is concerned with employing large molecules and the other with aggregates of atoms or molecules of colloidal size (simply called colloids). Both types of heavy particles present the problem of a mass distribution in the beam resulting in a loss of thruster efficiency. For colloids, however, reference 2 indicates that the mass distribution can be quite narrow.

At present, several methods of charging colloidal particles are being investigated. The method to be considered in this report is that of ionization by field emission of an electron. For reasons that are elaborated in the next section, only metallic aggregates, and, in particular, only those of the alkali group, are discussed in detail.

The field ionization method was studied in the investigation reported in reference 3 for the case of a gas of alkali atoms. Results were not encouraging, however, because of the excessive voltages required. This study was undertaken to determine if requirements might be less stringent for colloids than for atoms.

GENERAL DISCUSSION OF COLLOIDS

The structure of a colloidal particle is not clearly understood. It is highly improbable that any well-ordered arrangement of atoms prevails such as in a crystal. It is more likely that the colloid resembles a liquid droplet, within which the atoms are randomly arrayed.

The electronic structure of a colloid is similarly not understood. With the

assumption that the density of a colloidal particle is equal to its bulk density, it would be expected that metallic colloids contain unbound electrons; that is, electrons shared by all the member atoms of the aggregate. This follows from the nearness of neighboring atoms in a solid or liquid and also from the low ionization potentials of metal atoms. On the other hand, atoms with high ionization potentials, in the solid or liquid state, would be devoid of free electrons and, therefore, would be nonconducting. Removing electrons from such aggregates would hence be equivalent to ionizing the constituent atoms and would be a much more difficult task because of the higher ionization potentials involved.

To justify the hypothesis of free electrons, it may be pointed out that even a liquid metal is a good conductor despite the disorder of its atoms. An alternative statement of the free-electron condition is the fact that the energy levels of the valence electrons are not determined solely by the atoms in which they were originally bound, but by all the atoms in the colloid; that is, the allowed energy states must be determined by solving for the eigenvalues of the wave equation in which the electrons are shared by all the ionized atoms in the aggregate.

Because of the great difference in mass between electrons and atoms, the atoms can be considered as quasi-stationary, while the electrons move among them. Colloids, however, have no periodic structure, and thus an already difficult calculation for the energy levels becomes unmanageable.

With the assumption that these allowed energy states E in the droplet are somehow determined, Fermi statistics can be formally applied to obtain the following distribution function for the electrons:

$$f(E) = \frac{1}{\exp\left(\frac{E - E_0}{kT}\right) + 1} \quad (1)$$

(Symbols are defined in appendix A.)

The Fermi energy E_0 , however, would be both space and time dependent because of the random spacing of the metallic ions and the slowly changing interionic distances. These distances are slowly changing with respect to the movements of the electrons but are rapidly varying with respect to the motions executed by the colloid as a whole. Because of this, it would be expected that an average Fermi level could be assigned in equation (1). In lieu of detailed analysis and since this paper intends to give results indicative of the charging mechanism, the Fermi level is taken as equal to the Fermi level found in bulk materials. Elements with the smallest work functions will therefore be the easiest from which to remove electrons and thereby produce positively charged colloids. In view of this, the discussion is restricted to the alkali metals. Since the mass range of interest in thruster design lies between 10^4 and 10^6 atomic mass units (ref. 2), aggregates containing 10^3 and 10^4 atoms for each of the alkalis, lithium, sodium, potassium, cesium, and rubidium, are considered.

CALCULATION OF IONIZATION TIME

The electrons in a metallic colloid move under an average potential that is lower than the vacuum potential. At the surface, a potential "jump" to the vacuum level restrains the electrons from leaving the colloid. The probability of finding an electron at an energy greater than the vacuum level is exceedingly small at ordinary temperatures, as can be seen from the Fermi distribution given by equation (1).

An external electric field impressed on the colloid creates a potential barrier through which electrons can tunnel. The potential at the surface of the colloid is at the vacuum level and decreases linearly with a slope proportional to the field strength. The number of electrons n passing through this barrier per unit time depends on the number striking the barrier per second and on their transmission probability through the barrier. The reciprocal of this number is the average time required for one electron to tunnel through and is defined as the charging time τ :

$$\tau = \frac{1}{n} \quad (2)$$

If the field is in the x-direction and dn is the number of electrons within the colloid having the velocity v_x and a transmission probability $D(E_x)$, then it follows that

$$dn = \frac{2}{h^3} S f(E) v_x D(E_x) dp_x dp_y dp_z \quad (3)$$

where $2/h^3$ is the density of levels (two spin states per volume h^3 of phase space, h being Planck's constant); S is the effective surface area normal to the x-direction; and p_x , p_y , and p_z are the momentum components. The transmission probability is only a function of the energy in the x-direction.

The total number of electrons that will penetrate the barrier is then found by integrating over all possible momentums of the y- and z-components and over all positive values of the x-component:

$$\frac{1}{\tau} = n = \frac{2S}{h^3} \int_0^\infty dp_x v_x D(E_x) \int_{-\infty}^\infty \int_{-\infty}^\infty dp_y dp_z f(E) \quad (4)$$

When the last two integrations are carried out, there remains

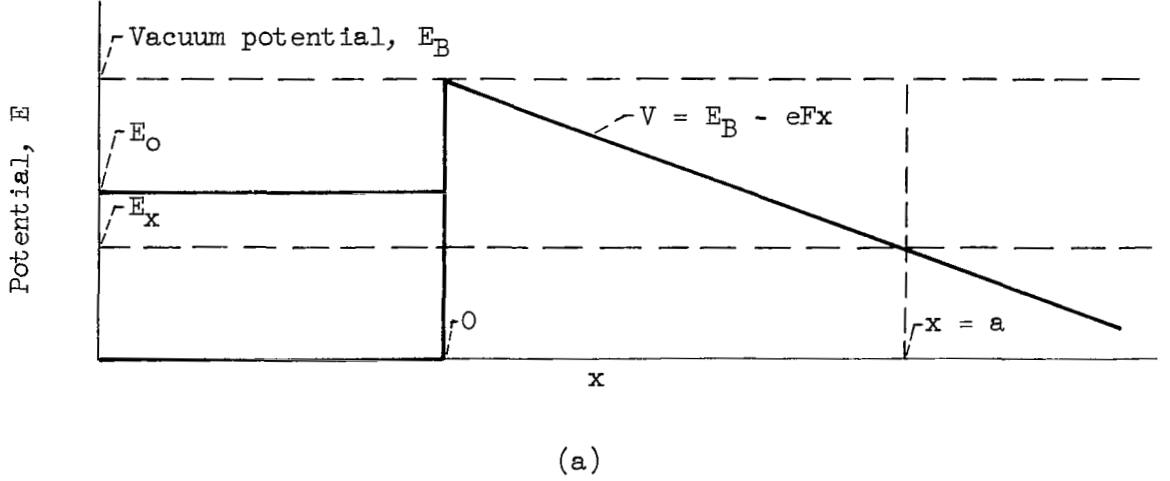
$$\frac{1}{\tau} = \frac{4\pi m_e k T S}{h^3} \int_0^\infty dE_x D(E_x) \ln \left[1 + \exp \left(\frac{E_0 - E_x}{kT} \right) \right] \quad (5)$$

Before this integration can be completed, it is necessary to find the transmission factor $D(E_x)$ as a function of energy. Because the potential is only a function of the x-coordinate, the wave equation is reduced to a one-dimensional

problem, and hence the W.K.B. (Wentzel, Kramers, and Brillouin) approximation (ref. 4) is applicable. Accordingly,

$$D(E_x) = \exp \left[- \frac{4\pi}{h} \int \sqrt{2m_e(V - E_x)} dx \right] \quad (6)$$

where the integration is carried out over the barrier. To simplify the calculation, $V(x)$ is approximated by a triangular barrier depicted in the following sketch:



The limit of the barrier at $x = a$ is seen from the sketch to be a function of both the applied electric field F and the energy E_x of the impinging electron. Carrying out the integration gives

$$D(E_x) = \exp \left[- \frac{8\pi(2m_e)^{1/2}}{3heF} (E_B - E_x)^{3/2} \right] \quad (7)$$

The ionization time is calculated with equation (7) substituted for $D(E_x)$ in equation (5):

$$\frac{1}{\tau} = \frac{4\pi m_e kTS}{h^3} \int_0^\infty dE_x \exp \left[- \frac{8\pi(2m_e)^{1/2}}{3heF} (E_B - E_x)^{3/2} \right] \ln \left[1 + \exp \left(\frac{E_0 - E_x}{kT} \right) \right] \quad (8)$$

Since this integration cannot be carried out in closed form, an approximation will be sought. Most of the contribution to the integral comes from below the Fermi level; therefore, the first simplification is to replace the upper limit by E_0 and to let

$$\ln \left[1 + \exp \left(\frac{E_O - E_X}{kT} \right) \right] \approx \ln \exp \left(\frac{E_O - E_X}{kT} \right) = \frac{E_O - E_X}{kT}$$

Next, by expansion of $(E_B - E_X)^{3/2}$ in a Taylor series about $E_X = E_O$ and by retention of only the first two terms, an integrable approximation is obtained:

$$(E_B - E_X)^{3/2} = \varphi^{3/2} \left[1 + \frac{3}{2} \frac{E_O - E_X}{\varphi} + \dots \right]$$

where φ , the work of function of the material, is equal to $E_B - E_O$.

Because the transmission factor rapidly converges to zero, the lower limit in equation (8) can be replaced by $-\infty$. With the substitution $\eta = (E_O - E_X)/\varphi$, the reciprocal charging time finally takes the form

$$\frac{1}{\tau} \approx \frac{4\pi m_e S}{h^3} \varphi^2 \exp \left[- \frac{8\pi (2m_e \varphi^3)^{1/2}}{3heF} \right] \int_0^\infty \eta \exp \left[- \frac{4\pi (2m_e \varphi^3)^{1/2}}{heF} \eta \right] d\eta$$

$$\frac{1}{\tau} \approx \frac{Se^2 F^2}{8\pi h \varphi} \exp \left[- \frac{8\pi (2m_e \varphi^3)^{1/2}}{3heF} \right] \quad (9)$$

Fowler and Nordheim's equation (ref. 5) for the current density emitted from a metal in a strong electric field shows the same functional dependence on the field strength as equation (9) although the method of derivation was not the same.

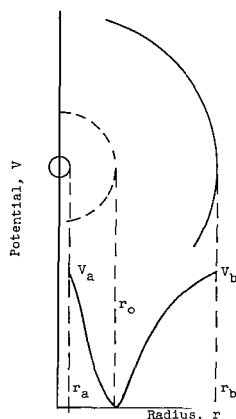
The effective surface area S in the field direction is taken to be the cross-sectional area of a spherical droplet of volume V :

$$S = \pi \left(\frac{3V}{4\pi} \right)^{2/3} = \pi \left(\frac{3N}{4\pi\rho_a} \right)^{2/3} \quad (10)$$

Where N is the number of atoms in the colloid and ρ_a is their number density, taken to be equal to the bulk density. Figure 1 shows a plot of charging time as a function of field strength for alkali-metal colloids containing 10^3 and 10^4 atoms. Table I contains the information used in calculating the curves shown in the figures.

IONIZATION PROBABILITY AND UTILIZATION EFFICIENCY

In order to produce the strong electric fields needed for ionization, it is necessary to use small geometries such as needle points or thin wires. An added effect of the field is produced by the neutral colloids being drawn into the regions of high field strength because of polarization forces. The polarization force afforded by the field of a cylinder is a longer range effect ($\sim r^{-3}$) than the field of a point ($\sim r^{-5}$). For this reason, the following geometry, consisting of three coaxial cylinders, is considered:



(b)

The central wire and the outermost cylinder are maintained at high potentials with respect to the grounded cylindrical grid. This arrangement forms a "potential well," as shown in the sketch.

It is assumed that a well collimated noninteracting beam of colloidal particles is introduced axially into one end of the tube. Because of the polarization forces, the particles are gradually drawn into the high-field region surrounding the central wire, charged, and repelled into the lower potential region. The charged particles oscillate in the potential well until they drift out of the tube. Once they are out, they can be introduced into a suitable thruster.

The classical expression for the polarizability α of a spherical conductor of radius R is

$$\alpha = 4\pi\epsilon_0 R^3 \quad (11)$$

This becomes for spherical colloids

$$\alpha = 3\epsilon_0 \frac{N}{\rho_a} \quad (12)$$

where N and ρ_a , as before, are the number of particles and the density of the droplet, respectively.

The potential energy of such a colloid in an electric field is then

$$U = - \frac{1}{2} \alpha F^2 = - \frac{3}{2} \epsilon_0 \frac{N}{\rho_a} F^2 \quad (13)$$

The field about the wire is radially symmetric and can be written in terms of the wire radius r_a and the surface field strength F_a :

$$F = F_a \frac{r_a}{r} \quad (14)$$

Since the beam is assumed to be well collimated, the angular component of the particle velocity is ignored, and the radial component is obtained from the equation of motion

$$\frac{dr}{dt} = - \left(\frac{3\epsilon_0}{\rho_a m^*} \right)^{1/2} \left(F^2 - F_1^2 \right)^{1/2} \quad (15)$$

where m^* is the atomic mass as given in table I(a) and F_1 is given by equation (14) at $r = r_1$, the initial distance from the origin.

The z-component is not affected by the field and is conserved through the tube even after ionization if no particle interactions occur.

The probability P that the colloid will emit an electron as it moves into the high-field region is given by

$$P = 1 - \exp \left(- \int \frac{dt}{\tau} \right) \quad (16)$$

where dt is the differential residence time from equation (15). In terms of the field strength, the integral in the exponent becomes

$$\int \frac{dt}{\tau} = F_a r_a \left(\frac{\rho_a m^*}{3\epsilon_0} \right)^{1/2} \int_{F_1}^{F_a} \frac{\frac{dF}{\tau \left(\frac{3}{2} F \right) F^2 (F^2 - F_1^2)^{1/2}}}{1} \quad (17)$$

The charging time τ in equation (17) is a function of an enhanced field strength, which has an average value of three-halves of the external field. This arises because of the distortion caused by the polarization of the colloid.

Initially, the colloids can enter the field at all possible values of F_1 ; that is, at any distance r_1 from the anode. The integrand, however, is a very

sensitive function of the field strength and rapidly vanishes away from the anode. For this reason, the initial field strength is neglected. This is equivalent to carrying out the calculation for a colloid starting at infinity. Colloids nearer the anode will, of course, also become ionized under the same conditions.

If the substitution

$$x = \frac{16\pi (2m_e \phi^3)^{1/2}}{9ehF_a r_a} r \quad (18)$$

is made, the integration takes the form of the well-known exponential integral $-Ei(-x_a)$, which is tabulated in almost all mathematical handbooks (e.g., ref. 6). In the expression

$$-Ei(-x_a) = \int_{x_a}^{\infty} \frac{e^{-x}}{x} dx \quad (19)$$

x_a is obtained from equation (18) with $r = r_a$. With these substitutions and definitions, equation (17) becomes

$$\int \frac{dt}{\tau} = - N^{2/3} \left(\frac{m^*}{4\pi\epsilon_0} \right)^{1/2} \left(\frac{3}{4\pi\rho_a} \right)^{1/6} \frac{e^2 F_a r_a}{h\phi} \left(\frac{9}{32} \right) Ei(-x_a) \quad (20)$$

The ionization probability (eq. (16)) is very sensitive to small changes in $\int \frac{dt}{\tau}$. These same changes, however, are reflected in equation (20) as a slight modification of the field strength (for a given radius); that is, the same order of magnitude field strength is needed to produce partial ionization of the beam as is used to achieve almost total ionization. In view of this, a condition will arbitrarily be chosen by letting $\int \frac{dt}{\tau} = 4$, which yields a probability of 0.982. The utilization efficiency at this degree of ionization of the beam should then be greater than 98 percent provided that the neutrals do not leave the tube before they come into the ionization region. Solving equation (20) for r_a with a value of 0.982 for the integral then gives the desired relation for the wire size as a function of the necessary field strength:

$$r_a = - \frac{128}{9} \frac{h\phi}{e^2 N^{2/3}} \left(\frac{4\pi\epsilon_0}{m^*} \right)^{1/2} \left(\frac{4\pi\rho_a}{3} \right)^{1/6} \frac{1}{F_a Ei \left[- \frac{16\pi (2m_e \phi^3)^{1/2}}{9ehF_a} \right]} \quad (21)$$

The potential V_a between the wire and the grounded grid cylinder of radius r_o is given by

$$V_a = F_a r_a \ln \frac{r_o}{r_a} \quad (22)$$

The field strength can be eliminated by combining equations (21) and (22) to obtain the voltage requirement as a function of wire radius. These are two of the more important design parameters plotted in figure 2 for grid radii of 10^{-3} and 10^{-2} meter.

To complete the discussion of utilization efficiency, it now remains to calculate the necessary cylinder length to allow each colloid to reach the ionization zone before leaving the tube.

The fraction of the colloidal beam having a velocity between v_z and $v_z + dv_z$ is given by

$$f(v_z)dv_z = 2 \left(\frac{Nm^*}{2kT} \right)^2 \exp \left[- \frac{Nm^*v_z^2}{2kT} \right] v_z^3 dv_z \quad (23)$$

Integrating from zero to v_z then gives the total fraction having velocities less than v_z :

$$f_{v_z} = 1 - (1 + v^2) \exp(-v^2) \quad (24)$$

where

$$v = \left(\frac{Nm^*}{2kT} \right)^{1/2} v_z \quad (25)$$

The time it takes a particle starting at the beam edge r_i to reach the anode ($r_a \sim 0$) should be made equal to its transit time through the tube traveling at a velocity v_z . Hence, by use of equation (15),

$$-\left(\frac{\rho_a m^*}{3\epsilon_o} \right)^{1/2} \frac{1}{F_a r_a} \int_{r_i}^0 \frac{r dr}{\left(1 - \frac{r^2}{r_i^2} \right)^{1/2}} = \frac{L}{v_z} \quad (26)$$

from which

$$L = \left(\frac{\rho_a 2kT}{3N\epsilon_0} \right)^{1/2} \frac{\nu r_1^2 \ln \frac{r_0}{r_a}}{V_a} \quad (27)$$

Table II gives the value of L at the atmospheric melting point of each of the alkalis at the following parameter values:

N	10^4
ν	1
$r_0 = r_1, \text{ m}$	10^{-2}
$r_a, \text{ m}$	10^{-5}
$V_a, \text{ v}$	10^5

If this value is represented by L^* , equation (27) can then be rewritten in terms of its more important parameters as

$$L = \nu \mu_1^2 L^*$$

where μ_1 is a nondimensional factor equal in magnitude to the radius of the beam expressed in millimeters, and ν , of course, gives the percent of beam i_{v_z} that will be ionized as determined from equation (24).

When the values of r_0 , r_a , and V_a were chosen, consideration was given to figure 2, which shows these values as occurring at realistic operating points. Except for variations in ν and μ_1 , equation (27) must be used as the basis for computing the tube length for other values of these parameters.

CHARGE AND CURRENT DENSITIES

The purpose of maintaining the outer cylinder at a high potential is to provide a sufficiently strong decelerating force on the charged colloids to keep them confined within the tube. In order to accomplish this, the potential must be at least as high as the potential on the emitting wire.

The charged particles are formed in the high-potential region around the wire. From figure 2, it is evident that, for practical emitter radii and operating potentials, each colloid would have an energy of the order of 10^5 electron volts if singly charged particles are assumed. The maximum density of neutral particles entering the tube is estimated to be of the order of 10^{15} per cubic centimeter. The interaction energy of two charged colloids at this density would then be of the order of 0.01 electron volt. The description of the motions

of the charged colloids within the tube then coincides with that of an ideal gas in which the particles move independently of each other but achieve thermodynamic equilibrium by virtue of the randomizing effect of the weak interactions between particles.

With the Boltzmann factor, the relative density of particles within the tube can be found. The potentials in the intercylindrical space, for the case where the outer cylinder is at the same potential as the wire, are given as

$$v(r) = \begin{cases} \frac{V_a}{\ln \frac{r_o}{r_a}} \ln \frac{r_o}{r} & r_a \leq r \leq r_o \\ -\frac{V_a}{\ln \frac{r_b}{r_o}} \ln \frac{r}{r_o} & r_o \leq r \leq r_b \end{cases} \quad (28)$$

In its most general form, the density can be written as

$$\rho = c \int \dots \int \exp \left(-\frac{H}{kT} \right) dp_1 \dots dp_{3N} \quad (29)$$

where $H(p_1 \dots p_{3N}, q_1 \dots q_{3N})$ is the Hamiltonian function of the conjugate momentums and coordinates p and q , and c can be considered to be a normalizing constant. Since the potentials depend only on the coordinates, the integrations over the momentums contribute a constant factor and the density is a function of the coordinates. In particular, because of equation (28), the density is a function of the radial coordinate alone:

$$\rho(r) = \rho(r_o) \exp \left[-\frac{eV(r)}{kT} \right] \quad (30)$$

where $\rho(r_o)$ is a constant equal to the maximum density that occurs at $V = 0$, that is, at $r = r_o$. In terms of the potentials of the two regions, as given in equation (28), the normalized density is then

$$\rho = \begin{bmatrix} \rho(r_o) \exp \left(-\frac{eV_a}{kT} \frac{\ln \frac{r_o}{r}}{\ln \frac{r_o}{r_a}} \right) = \rho(r_o) \left(\frac{r}{r_o} \right)^{\frac{eV_a}{kT} \frac{1}{\ln \frac{r_o}{r_a}}} & r_a \leq r \leq r_o \\ \rho(r_o) \exp \left(-\frac{eV_a}{kT} \frac{\ln \frac{r}{r_o}}{\ln \frac{r_b}{r_o}} \right) = \rho(r_o) \left(\frac{r_o}{r} \right)^{\frac{eV_a}{kT} \frac{1}{\ln \frac{r_b}{r_o}}} & r_o \leq r \leq r_b \end{bmatrix} \quad (31)$$

The quantity eV_a/kT needed in equations (31) requires a knowledge of the ion gas temperature. Since the initial conditions are approximately known, the effective temperature can be calculated. The average energy per emitted particle is approximately eV_a , while from the ideal gas equation, the average energy is $\frac{3}{2} kT$. Equating these terms gives the desired relation

$$\frac{eV_a}{kT} = \frac{3}{2} \quad (32)$$

A field-emission ionizer, such as the one described, can never operate under space-charge-limited conditions, for this implies that the field at the emitter vanishes. It is not clear what the exact saturation level is, but it is obvious that the field does not have to be suppressed very greatly to cause a notable increase in the charging lifetime and thus end field emission. If it is assumed that an anode field-strength suppression of $10^{-2} F_a$ is the allowable maximum, an estimated upper limit to $\rho(r_o)$ (appendix B) is

$$\rho(r_o) = \frac{10^{-2} V_a \epsilon_o \left(\frac{3}{2 \ln \frac{r_o}{r_a}} + 2 \right)^2}{er_o^2} \quad (33)$$

The average flux density, \bar{j}_N out of the tube end, with an assumed mean drift velocity \bar{v}_z , is then equal to

$$\bar{j}_N = \frac{\bar{v}_z}{2} \int_{r_o}^{r_a} \rho(r) r \, dr \quad (34)$$

$$\bar{j}_N \cong \bar{v}_z \rho(r_o) \left(\frac{3}{2} \frac{1}{\ln \frac{r_o}{r_a}} + 2 \right)^{-1} \quad (35)$$

The mean velocity of a molecular beam in the mass range being considered ($Nm^* \sim 10^{-22}$ kg), at a temperature of the order of 1000° K, is approximately 7 meters per second, as computed from

$$\bar{v}_z = \frac{3}{4} \left(\frac{2\pi kT}{Nm^*} \right)^{1/2} \quad (36)$$

The value of \bar{j}_N calculated at a likely design point, for instance, $V_a = 10^5$ volts, $r_a = 10^{-5}$ meter, and $r_o = 10^{-3}$ meter is therefore of the order

of 10^{18} per square meter per second. This figure, if multiplied by the electronic charge e gives a maximum current density of 0.144 ampere per square meter from this device in thermal equilibrium; that is, if no field is used to draw the charged colloids out of the tube. This figure also represents the necessary input of neutral colloids from the particle generator, in order to achieve maximum charging efficiency. More than this would cause saturation and a resulting loss in ionization efficiency. The use of even a moderate electric field applied axially along the cylinder, however, can greatly increase the current density.

CONCLUDING REMARKS

The field emission method of producing positively charged colloids is, on the basis of net power per charge, one of the most efficient ways of charging.

The propellant-utilization efficiency, as pointed out in the text, could be made to approach unity in a cylinder of reasonable length. This, of course, is contingent on how well the colloid generator can form aggregates from the monatomic propellant.

At present, the necessary relation between the applied potentials and the emitter radii required for this charging method is somewhat difficult to achieve, but it is likely that in the future monocrystalline filaments and/or high-voltage sources will be available. With the advent of such thin filaments, it is possible that the required emitter potential will be reduced enough so that a magnetic field can be used to turn the charged colloids, and thus the intermediate grid could be eliminated. This is desirable since the grid is located in the region of greatest charge density and is subjected to severe impingement.

Lewis Research Center
National Aeronautics and Space Administration
Cleveland, Ohio, August 13, 1963

APPENDIX A

SYMBOLS

a	potential barrier end point
c	constant
D	transmission probability
E	energy level or state
E_B	vacuum potential
E_O	Fermi energy level
E_x, E_y, E_z	translational energy in x-, y-, and z-directions, respectively
e	electronic charge
F	electric field strength
F_a	electric field strength at anode
F_i	electric field strength at r_i
f	Fermi distribution function; velocity distribution function
f_{v_z}	fraction of particles having velocities up to v_z
H	Hamiltonian function
h	Planck's constant
\bar{j}_N	average flux density
k	Boltzmann's constant
L	cylinder length
L^*	specific cylinder length, and table following it defined by eq. (28)
m_e	electronic mass
m^*	atomic mass
N	total number of constituent atoms in colloid
N_O	Avogadro's number

n	number of electrons passing through potential barrier per sec
dn	differential number of electrons passing through potential barrier per sec
P	ionization probability
p	momentum
p_x, p_y, p_z	momentum components in x-, y-, and z-directions, respectively
q	coordinate
R	effective colloid radius
r	radius
r_a	anode radius
r_b	outer cylinder radius
r_i	initial radial distance from anode
r_o	grid radius
S	effective surface area presented to tunneling electrons
T	temperature
t	residence time
U	potential energy
V	potential function
V_a	voltage at anode
V_b	voltage at outer cylinder
v_x, v_y, v_z	velocity components in x-, y-, and z-directions, respectively
\bar{v}_z	average velocity in z-direction
x	dummy variable in eq. (20)
x_a	defined by eq. (19) at $r = r_a$
x, y, z	Cartesian coordinates
r, θ, z	cylindrical coordinates

α	polarizability
ϵ_0	vacuum dielectric constant
η	dummy variable
μ_i	nondimensional beam radius
ν	variable defined by eq. (27)
ρ	density function
ρ_a	number density of atoms in colloid
ρ_w	density of water at 4° C
σ	surface variable
τ	charging time
ϕ	work function

Subscripts:

i	initial
$1 \dots 3N$	degrees of freedom

APPENDIX B

SUPPRESSION OF ANODE FIELD STRENGTH

The field strength at a distance r due to a cylindrically symmetric cloud charge of density $\epsilon_0 \rho(r)$ and a central wire conductor having a surface charge density of $\epsilon_0 F(r_a)$ is

$$F(r) = \frac{1}{2\pi\epsilon_0 r} \left\{ 2\pi\epsilon_0 r_a F(r_a) + e \int_{r_a}^r \rho(r) 2\pi r dr \right\} \quad (B1)$$

The wire is held at a constant potential V_a with respect to the grounded grid at r_0 . Hence, this voltage is given alternately by equations (22) and (B2):

$$V_a = - \int_{r_0}^{r_a} F(r) dr \quad (B2)$$

If the appropriate expression for $\rho(r)$ from equation (31) is used and the indicated integrations are carried out, the effect of the space charge on the wire field strength is found to be

$$F_a - F(r_a) = \frac{\epsilon_0 \rho(r_0) r_0^2}{\epsilon_0 r_a \ln \frac{r_0}{r_a} \left[\frac{3}{2 \ln(r_0/r_a)} + 2 \right]^2} \quad (B3)$$

Terms involving powers of r_a are ignored since $r_0 \gg r_a$.

If a field-strength suppression factor of

$$F_a - F(r_a) = 10^{-2} F_a \quad (B4)$$

is assumed, the maximum density becomes

$$\rho(r_0) = \frac{10^{-2} V_a \epsilon_0 \left[\frac{3}{2 \ln(r_0/r_a)} + 2 \right]^2}{e r_0^2} \quad (B5)$$

in which equation (22) was again used.

TABLE II. - CYLINDER LENGTH AT
MELTING POINT

Element	Melting point at 760 mm Hg, °K	Specific cylinder length, $\frac{L^*}{m}$
Lithium	459.2	3.25×10^{-1}
Sodium	370.7	2.19×10^{-1}
Potassium	335.5	1.48×10^{-1}
Cesium	301.7	3.54×10^{-2}
Rubidium	311.7	4.10×10^{-2}

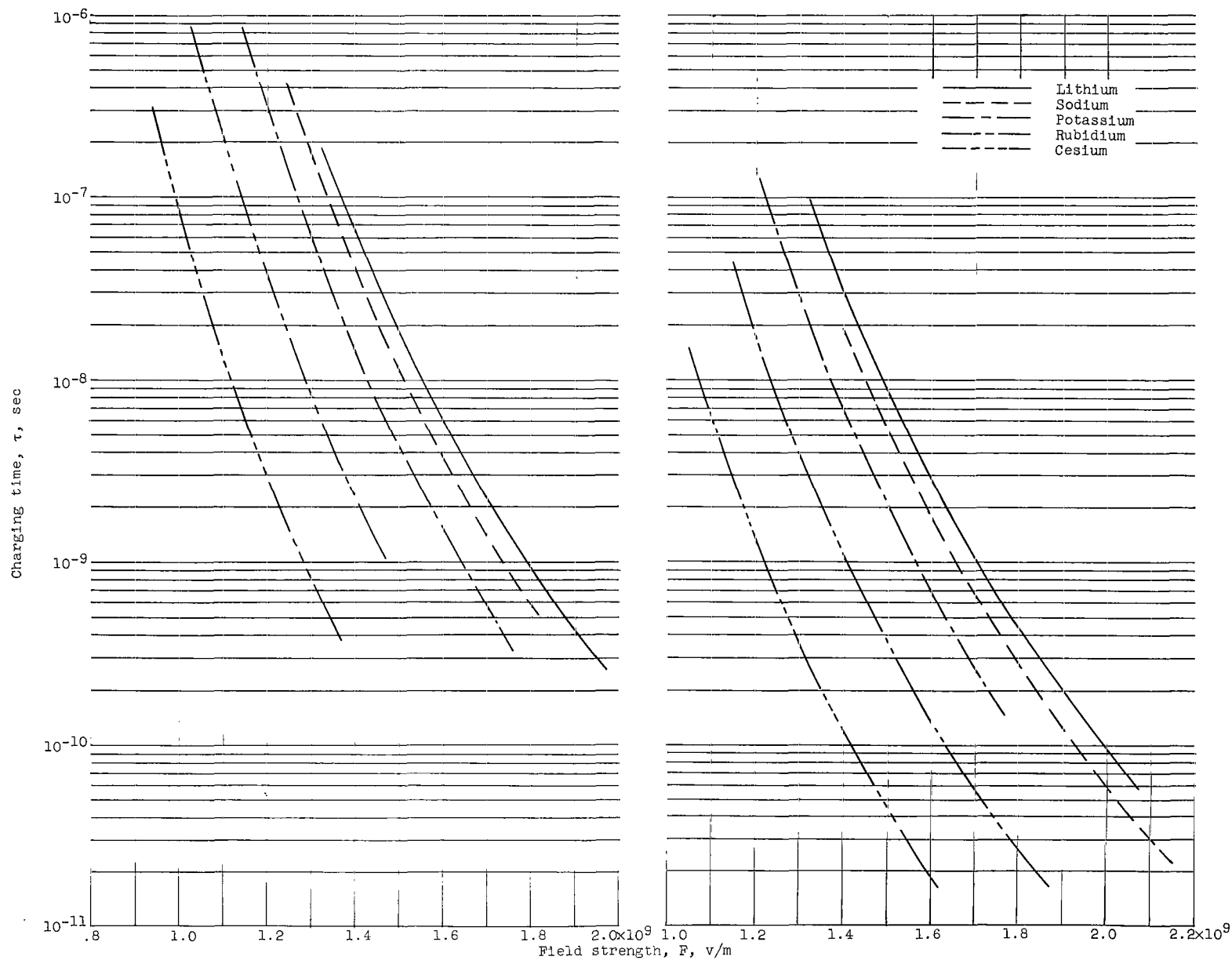
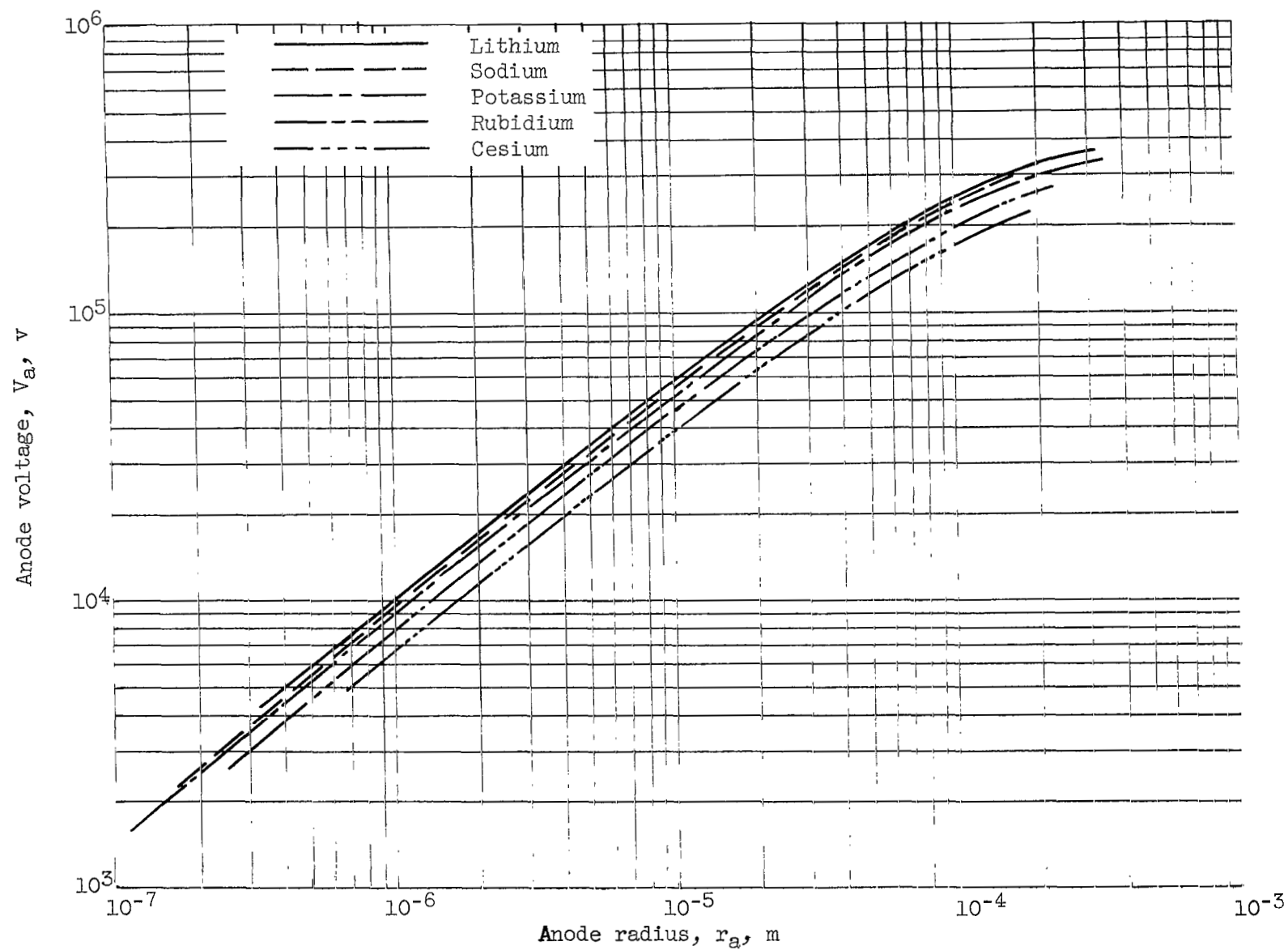
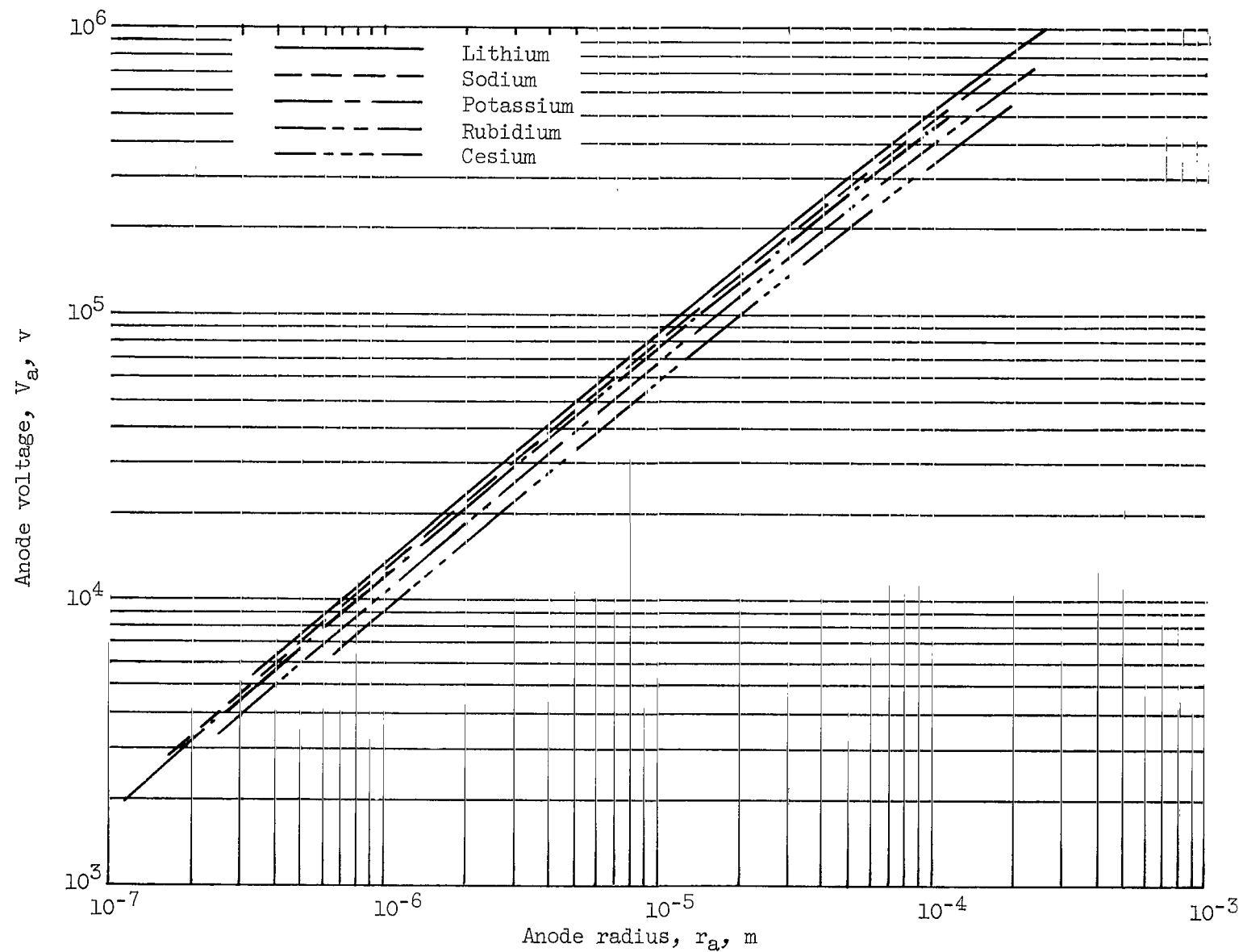


Figure 1. - Charging time as function of field strength.



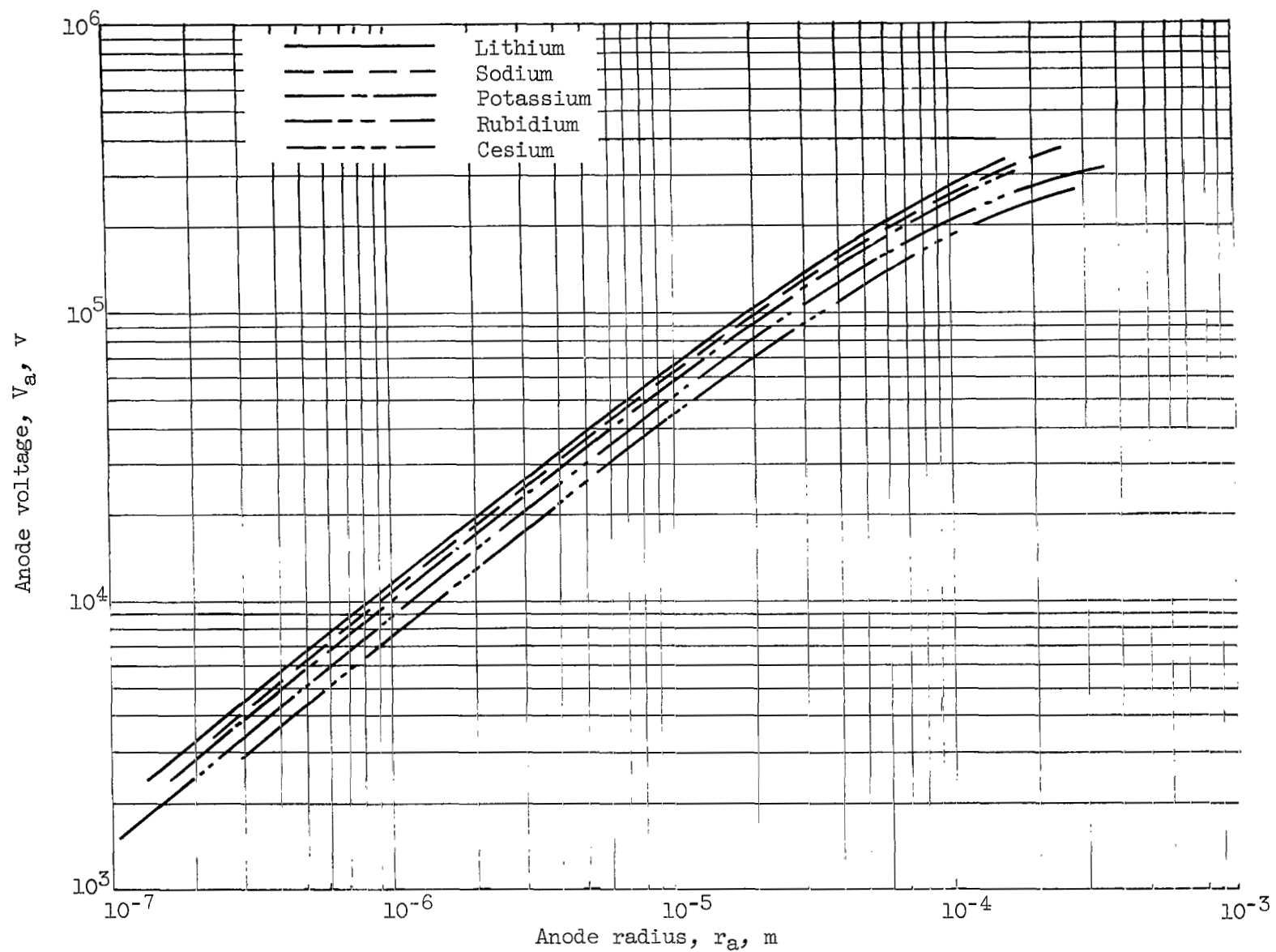
(a) Atom density, 10^4 atoms per colloid; grid radius, 10^{-3} meter.

Figure 2. - Anode voltage as function of anode radius.



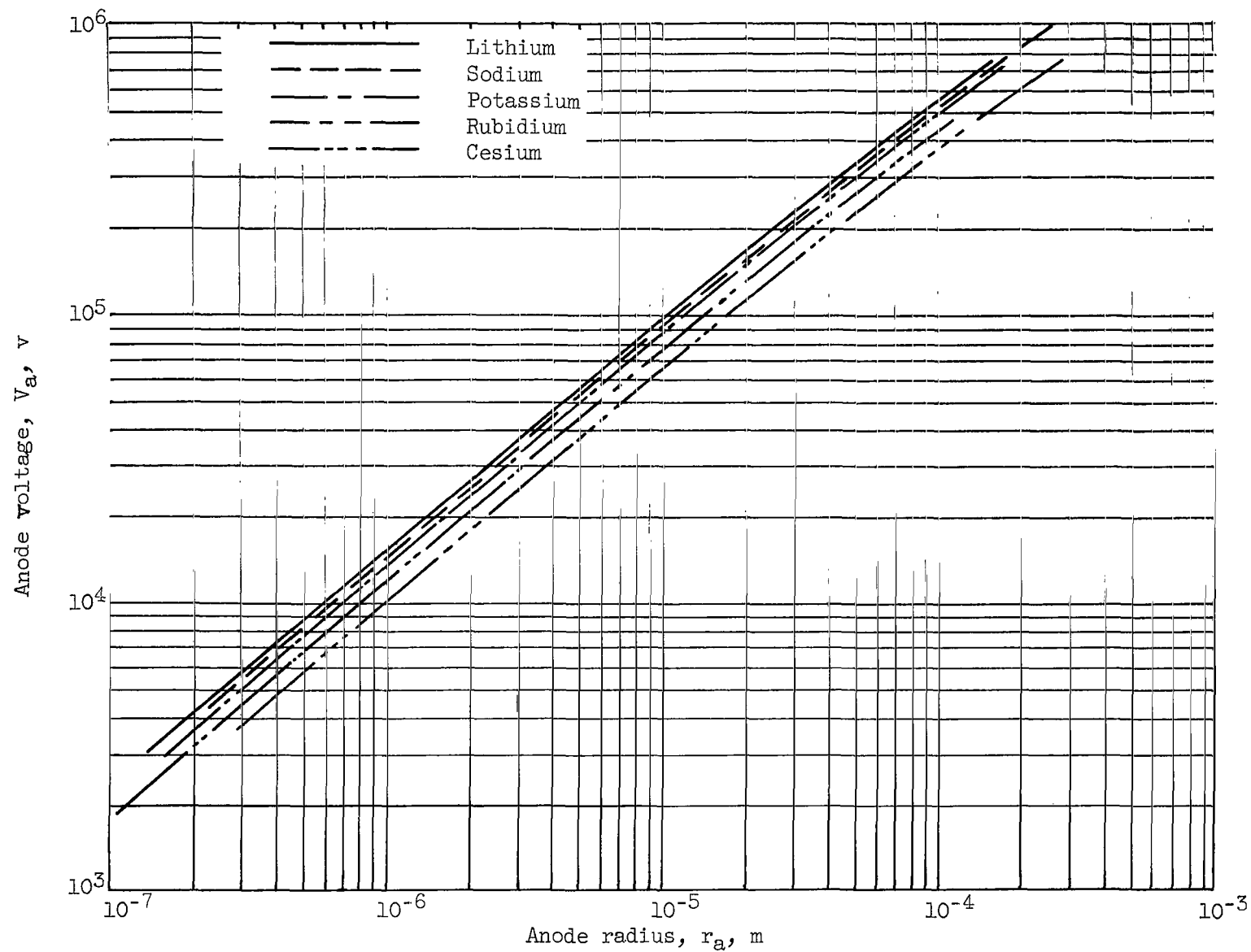
(b) Atom density, 10^4 atoms per colloid; grid radius, 10^{-2} meter.

Figure 2. - Continued. Anode voltage as function of anode radius.



(c) Atom density, 10^3 atoms per colloid; grid radius, 10^{-3} meter.

Figure 2. - Continued. Anode voltage as function of anode radius.



(d) Atom density, 10^3 atoms per colloid; grid radius, 10^{-2} meter.

Figure 2. - Concluded. Anode voltage as function of anode radius.